Drawing (Inferences) Outside the Lines: Dimensionality in Congress

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August 19, 2010

Abstract

There exists a general consensus that much of the behavior of political elites, and thus the preferences of those they represent, can be adequately represented a one- or two- dimensional space, the primary dimension of which is characterized as the left-right/liberal-conservative spectrum. However, these statistical findings are to some extent driven by assumptions that precede the estimation of the space. In this paper, we conduct several simulation experiments using various modeling assumptions regarding the voting behavior of members of Congress. We consider models where voting is a function of purely ideological, purely distributive, or mixed preferences, and scale the resulting roll-call matrices.

Our main finding is that while a truly low-dimensional ideologically-driven congress can be explained with only a few dimensions, the estimation of a low-dimensional space does not necessarily imply a truly low-dimensional world. Rather, party polarization, large numbers of ideological dimensions, and distributive politics will all tend to generate similar findings.

A previous version of this paper was presented at the 2009 Annual Meeting of the Southern Political Science Association in Atlanta, GA.

1 Introduction

The dimensionality of politics is a central concern to political science. Is each policy conflict a unique event that has no bearing on other public debates, are all conflicts part of a grand ideological struggle regarding how "liberal" or "conservative" government policy should be, or is it something in between? The answers to these questions have significant implications for almost all areas of research. Although literature has largely focused on theoretical models of and data from Congress, the structure of political debate bears on concepts of representation, public opinion formation, and democratic politics in general. If we are to develop scientific theories of the dynamic process that connects voter opinions with policy outputs and test them using meaningful measures, we must understand the basic structure of political debate.

The substantive importance of the dimensionality of policy conflict is reflected in the wide attention it has received in recent years. What would seem to be a rather abstract and arcane methodological issue has recently been studied by many of the leading scholars on Congress (e.g., Poole and Rosenthal 2007; Crespin and Rohde 2007; Roberts, Smith and Haptonstahl 2009) and public opinion (e.g., Stimson 2004). For the most part, the debate surrounds the dominant belief that policy conflict can be accurately characterized by very few dimensions. Most often the claim is that there are "one-and-a-half" issue dimensions (e.g., Poole and Rosenthal 2007).

In this paper we argue that widely-held conclusion that the scope of policy conflict in the United States is low dimensional cannot be justified by the statistical methods most commonly deployed to support this claim. Our argument comes in two parts. The first is purely methodological. Exploratory statistical procedures – whether principal components factor analysis, W-Nominate (Poole and Rosenthal 2007), optimal classification (Poole 2005), Bayesian item response theory (Clinton, Jackman and Rivers 2004), or any other variant (Heckman and Snyder Jr 1997) – do not by themselves provide evidence for or against the low-dimensionality hypothesis.

This is not to say that the estimates generated by these procedures are invalid or in some way flawed. *Given* that (i) the assumptions of these models are met and (ii) there is sufficient data, *then* standard diagnostics will inevitably lead researchers to the correct conclusion regarding the underlying dimensionality of the data. However, the converse of this statement does not hold. Scree plots, eigenvalues, and other approaches for diagnosing dimensionality with these scaling methods tell us nothing about the appropriateness of the statistical assumptions, nor the true number of dimensions. In addition, the number of roll-call votes available for analysis in the typical Congress makes it impossible to sustain "estimation" of a large number of dimensions, even when such is the true state of affairs.

The second part of our argument is more theoretical and substantive in nature. We argue that there are many plausible circumstances under which researchers may be led to falsely conclude that political conflict is inherently low-dimensional. Indeed, we might even go further in arguing that such circumstances are common, and the observation of low dimensionality is non-diagnostic. The specific circumstances on which we focus are, first, when there are bi-modal distributions of preferences and, second, when there are stable factions of members engaged in distributive politics (Baron and Ferejohn 1989). As we show through Monte Carlo simulations below, in both of these situations standard scaling techniques are likely to lead to incorrect inferences regarding the dimensionality of political conflict.

In the next section, we elaborate upon our argument. In the following sections, we use Monte Carlo simulations of congressional roll-call voting to evaluate the ability of standard scaling techniques to accurately recover the pre-specified dimensionality of the space. In Section 3, we begin by assuming standard non-strategic policy voting, showing that these procedures make significant errors in high-dimensional settings ($N > 8$) due to constraints imposed simply by the size of the roll-call record data set. In Section 3.2, we show that these problems are exacerbated considerably when there are bi-modal distributions of preferences amongst members, akin to a polarized Congress (whether polarized by parties or otherwise).

In Section 4, we turn to the Baron and Ferejohn (1989) bargaining model of Congress – an inherently high-dimensional setting. We show that when collections of legislators form even modestly stable coalitions in this distributive setting, standard scaling techniques will recover a single policy dimension. In Section 5, we then briefly examine the impact of mixing distributive and policy concerns in the behavior of members. We again find a marked tendency for scaling procedures to mis-estimate the number of latent dimensions. We conclude with a discussion of the implications of our findings.

2 The limitations of scaling

For nearly a generation, congressional research has advanced empirically on scaling estimates based on roll-call data. Indeed, one of the first systematic statistical advances in Political Science was the creation of the Rice index (Rice 1925) which analyzed patterns in roll-call voting data. Various multidimensional scaling techniques have been invented more recently, with Keith Poole and Howard Rosenthal's extensive work, yielding the Nominate procedure, currently the most well known and most extensively used (Poole and Rosenthal 1997, 2007). Their procedure is, indeed, a significant advance that fully deserves its fame and utilization. Yet, Poole-Rosenthal's scaling technique(s) (hereinafter, PR) and their numerous cousins are no more appropriate for answering every question in congressional research than are, say, surveys appropriate for answering every question in the study of voting behavior.

Like all techniques, PR and its kin develop on the basis of specific assumptions that condition the scope of applicability. Like all techniques that are extensively used, they can be misused. Further, because they are based on a specific data source – recorded votes cast on the floor of the two chambers – the data themselves limit the range of applicability of, and the nature of inferences able to drawn from, W-Nominate scores or their kin.

Our major claim is that the observation of a small number of dimensions resulting from application of such scaling procedures to data such as congressional roll-call data is insufficient to support the inference that the true number of dimensions is indeed small. As we will see, the results of our Monte Carlo simulations are discomforting in that they show that observing "one-and-a-half" dimensions is consistent with the true number of dimensions being small *and* being large. Moreover, the simulations show that these misleading inferences can occur under many reasonably likely, and theoretically plausible circumstances.

2.1 How informative are scaling estimates?

Our starting point is that there is no research design that makes the number of dimensions a *derivation* from theory. No extant theories predict empirical patterns that would in some sense "prove" that a certain number of dimensions is appropriate, nor would they sustain inferences about the number of dimensions. This includes the theories that are assumed to hold by commonly used scaling procedures themselves (Poole and Rosenthal 2007).

PR explicitly embed their scaling procedure within a spatial modeling framework. The dimensionality is defined simply as an n-dimensional real valued function that take $\mathcal{R}^n \rightarrow \mathcal{R}$. However, their stated substantive understanding is that dimensions represent the policy arenas over which citizens have preferences and on which voters evaluate candidates, officeholders, and government actions. Candidates take positions ("platforms") in this space to win votes.[1]

In office, incumbents are supposed to be taking action to pass laws or otherwise achieve policy outcomes. Of course, this is so because there is a presumed (simple) mapping from citizens' preferences to the candidates' policy platforms, and onto the officeholders' votes. Thus, in the common spatial model, members of Congress have preferences over policy dimensions, and these preferences are often reelection-induced policy preferences.

One strength of the spatial model is that these are, or at least can be, exactly the same policy dimensions and choices inside the legislature as in the electoral arena. In short, the policy space is the tapestry on which the full set of players in democracy have preferences and on which they condition their choices and actions. Getting the space right is therefore a central issue in the application of these models and the scaling techniques based on them.

Yet, as noted above, spatial models have very little to say about how we are to go about defining

[1]They may also take positions to realize their own preferences over the dimensions, just as if they were citizens themselves.

the space. It is assumed, not derived from theory. The theory requires only that it be real valued functions and measured via a Euclidean metric, or perhaps some other Minkowski p-metric. The theory is not very constraining. This is a good thing for theorizing. Less constraining assumptions mean that results derived from them are more general. It does, however, place a greater burden on the empirical research.

PR are explicit in stating that their estimation technique derives from this, indeed a specific form of this, spatial model. They assume, for instance, that all actors vote sincerely. It is through these assumptions that their estimates are derived. If this set of assumptions holds, they demonstrate that their procedure will produce good estimates. However, these are sufficiency results, not proof of necessity.

Their assumptions are not necessary. Other conditions – perhaps a very different set of assumptions – might yield exactly the same results. It is this fact that is the focus of this paper. Its relevance is that estimates derived from PR analyses of roll-call data are perfectly reasonable *if their assumptions are correct*. However, if these assumptions are not valid there may be other superior alternatives.[2] Indeed, we will see that there are broad categories of cases in which their assumptions are correct (meaning their assumptions are built into the Monte Carlo simulations) and their procedure still yields an "incorrect" number of dimensions.

We can infer from assumptions to their conclusions, but we cannot infer back. It is not a $1 \rightarrow 1$ relationship. Thus, for example, finding that there are two dimensions that fit the observed data well means that the true space could be two-dimensional, but (as we will show in our simulations) it might also mean that there are many more dimensions.

2.2 Plausible alternatives to the simple spatial model

In the wake of the 2002 midterm elections, the U.S. House of Representatives reconvened for lame-duck session. One goal of the session was to pass a conference report on a bankruptcy reform measure (HR 333) aimed at protecting creditors by making bankruptcy more difficult. Previous reform

[2] Clinton and Meirowitz (2003) make a similar argument.

efforts were blocked by Senate Democrats and President Clinton, and passage was a major aim of the Republican leadership. However, the bill had received stiff opposition from anti-abortion Republicans due to a single Senate amendment[3] aimed at preventing anti-abortion protesters from filing for bankruptcy in order to avoid paying court-ordered judgments.

With the support of Henry Hyde (a leading pro-life advocate in the House), the Republican leadership brought the rule for the bill to the floor. However, other pro-life advocates continued to rally against it. CQ notes that:

> Republicans were caught in a tug of war between their leadership ... and their conservative colleagues.... About 20 minutes after the vote began, Speaker J. Dennis Hastert, R-Ill., who by custom does not vote, cast his vote in favor of the rule, creating a tie at 204. That was the closest Republican leaders got to winning. After GOP leaders acknowledged that their bid was doomed, many Republicans who had supported the rule out of loyalty to their leaders rushed to change their vote (CQ 2002, C-10).

In the end, the rule was rejected, 172-243 with 87 pro-life Republicans joining 155 Democrats to effectively kill the bill. Indeed, the controversy over this single abortion provision stalled reform for another three years.

Many have interpreted the scaling estimates of Congress as supporting the inference that the U.S. Congress is approximately one-dimensional, or more generously, one-and-a-half dimensions.[4] The point most make, of course, is that substantively, the pattern of roll-call votes is so highly structured that something close to one dimension is sufficient to describe them very well. And with that we do not disagree. Our first problem is that the question is not just whether the data are sufficiently patterned, but also *why* is that so. Our second problem is with the practice of taking these results and inferring backwards that it is a reasonable estimate of the true policy space on which those votes were determined.

[3]The amendment was offered by Democratic Senator Charles Schumer (NY), and had been a sticking point in previous versions of the bill.

[4]Technically, even that is not true. Using the standard definition of what makes a dimension empirically a dimension, PR never get fewer than three significant dimension and often get a larger number, up to nine. As we discuss below, lacking a basis of inference, we cannot actually make claims about "statistically significance" from such estimates.

What incidents like the one described above reveal is that there many be many factors that go into a single roll-call for an individual member. Did HR 333 suddenly become more "conservative" or "liberal" 20 minutes into the vote? Or was there something else at work that was structuring member behavior that broke down under an unusual set of pressures? And in how many less unusual circumstances do such forces serve to structure members' votes for reasons totally unrelated to "preferences" or ideology? The answer is that, in many cases, we cannot know. Moreover, as we show in the Monte Carlo simulations below, given the presence of these kinds of non-spatial factors, there is very little one can conclude from the observation of one or on-and-a-half dimensionality in the roll-call data.

To be more specific, we consider three potentially different explanations for member behavior in this paper. We have written so far about the standard spatial model (Black 1948; Downs 1957). In that model, citizen preferences determine where candidates stand for election, what they do in office, and so on. In this model, a nearly one-dimensional space would resemble a single liberal-conservative ideological struggle.

A second alternative is that it might be parties that determine what divides members. No matter how many true dimensions there are, if the two parties are sharply polarized, it takes exactly one dimension to describe the differences. It is thus an artifact of our two-party system, at least when combined with sufficient polarization, that creates the nearly one dimensional appearance. In this account, when parties were internally divided (as in the 1950s and 1960s) and there was considerable overlap between them, it would take more than a single dimension to describe the data. As the two parties polarized and the overlap shrank, a single dimension became increasingly sufficient. Note that this account does not specify why parties were divided in Congress and are now polarized. It only claims that if there is partisan polarization, for whatever reason, roll-call votes will appear approximately unidimensional.

There is a third alternative. Perhaps the second most important contribution to the study of democracy are theories based on distributive politics (e.g., Shepsle 1979; Shepsle and Weingast 1987; Weingast and Marshall 1988) and the Baron-Ferejohn bargaining model (1989) that assumes preferences (and policy alternatives) are distributive. In such models, the space is not the same policy dimension as in the Black-Downs spatial model. Rather it is pork, the allocation of concentrated benefits (in the theory, typically to a single district/constituency) with costs being dispersed across all districts (as taxes being shared equally at 1/435th per constituency in the House).

In this version of Baron-Ferejohn politics, and if it were only the House at issue, the space would be 434-dimensional. In the Senate, it would be either 99- or 49-dimensional (depending upon how one treats the two Senators from the same state). In any event, pork-barrel politics is necessarily of high dimensionality. How one might consider partisan politics in this regard is a further set of refinements that might shape the number of dimensions in play, but is unlikely (and is not, in the way we do here) going to reduce the number of true dimensions to an actually small number.

In sum, our central conclusion is that we will observe that it is possible to get such estimates as one-and-a-half dimensions from spatial models when the true number of dimensions is small or large, with two parties that are divergent and polarized, and also with pork barrel politics. An estimated space of one-and-a-half dimensions can arise from many different configurations. We claim that "one-and-a-half-dimensions" can follow from several accounts that have considerable theoretical and empirical plausibility.

2.3 Of Elbows and Eigens

In the Monte Carlo simulations below we will be generating roll-call matrices based on various assumptions about MC's preferences and behaviors. For each imagined congress, members will follow a simple set of rules to cast multiple votes that will then be analyzed. Our primary interest will be establishing the dimensionality that standard multidimensional scaling techniques would produce. Therefore, it is worth taking a moment to briefly discuss various approaches for identifying dimensionality.

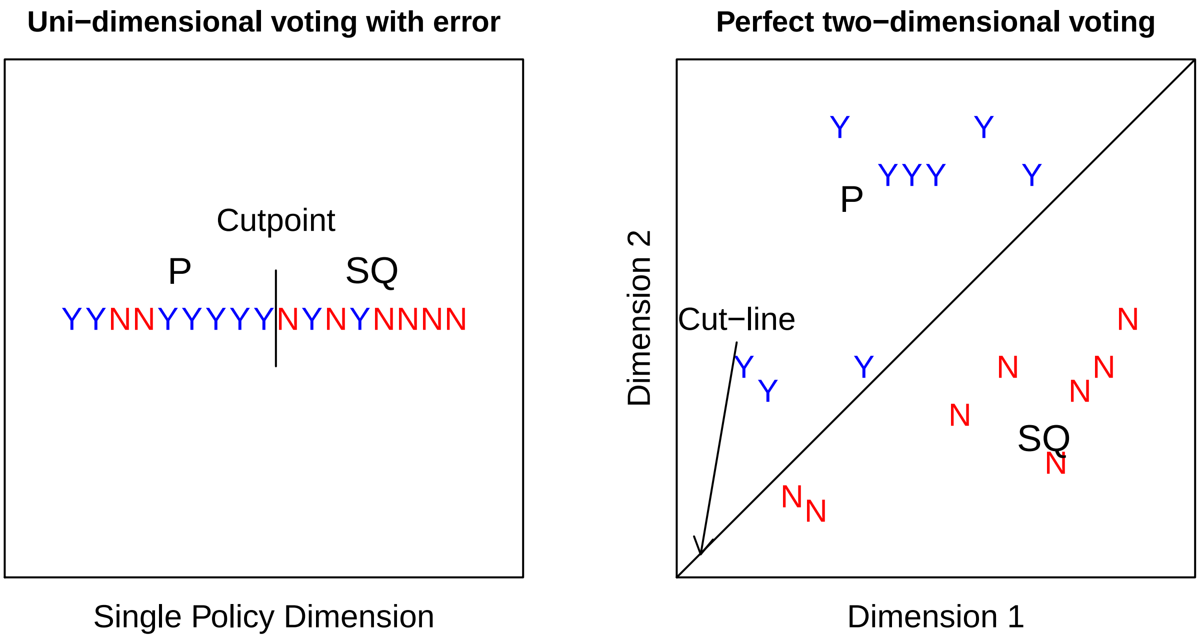
Ideally, what we would like is some theoretical model of dimensionality itself. We would like to be able to deductively derive from some set of assumptions an empirical regularity or pattern that would reveal the true underlying dimensionality. And this is often how scaling procedures are treated. However, scaling procedures, at least the currently available methods, do not do this. Rather, they take the dimensionality as a maintained assumption. An inference, to the extent that this can be said to be an inference, is then made by saying, if we compare the estimated model with a maintained assumption that there is a single dimension to one that assumes there are exactly two dimensions, and those, in turn, to the model that assumes there are actually and exactly three dimensions, one of those models seems like the most "adequate" for the task at hand. But that just a judgment call, not a formal test.

The adequacy of the model is determined by the number of observations we are comfortable with describing as external to the model or as "random." In order to account for all of the observed data in the roll-call record for n-legislators perfectly we would need $n - 1$ dimensions (or nearly that). The acceptance of fewer dimensions is based on a judgment call about the acceptable number of errors we are willing to chalk up to error. As shown in Figure 1 it is always possible to remove additional "noise" by the continued addition of dimensions. The left panel of Figure 1 shows a simplified example of a one-dimensional world for one roll-call. As can be seen, there are several errors in the predictions of the models based on the estimates of member positions. The right panel, however, shows how it is always possible to add an extra dimension that will allow the model to perfectly predict this particular roll-call without altering the estimates on the first dimension.

In some cases, such as non-metric, multidimensional scaling,[5], the problem is purely inductive. One is given a matrix of similarities between the items being scaled. The "model" consists of a single assumption - the greater the similarity, the closer the items should be in a Euclidean-based geometry (or some other geometric metric). There is no assumption at all about the larger world, either in the sense of a substantive theory or in the sense of a data generating process. This is therefore the most inductive approach possible. Resulting configurations have no hypotheses being tested, nor is there a theory of the random-sample-from-a-universe sort by which to infer standard errors or make

[5]See Kruskal (1964*a*,*b*); Shepard (1966). For applications to political science see Weisberg and Rusk (1970) and Aldrich and Sparks (2010).

Figure 1: Misclassifications as error or additional dimensions



It is always possible to add dimensions to better predict the data.

inferences to a population.

Others scaling techniques, such as factor analysis (think exploratory factor analysis or principal components analysis) do not have a substantive theory being "plugged" in to test hypotheses (rather like confirmatory factor analysis intends), but it does make assumptions about the larger universe from which standard errors and the like can, in principle, be derived for at least some parameters. Poole-Rosenthal scaling, farther along this continuum, makes a very large set of assumptions about both the nature of congressional decision making (all MCs have on n-dimensional, single-peaked, symmetric utility functions, voting is purely sincere rather than strategic, etc.) and about the data generation process. However, as we explore in the next sections, when the behavioral and distributional assumptions of these scaling models are violated, their ability to support inferences fails.

Nonetheless, various scholars have offered heuristics or rules of thumb for identifying the appropriate number of dimensions. The two most prominently used are Kiaser's eigenvalue-greater-than-one rule (Kaiser 1960) and the so-called "elbow-test" proposed by Cattell (1966). Each of these heuristics is designed to help scholars make a *judgment* regarding whether "enough" of the structure of the data has been explained by a specific number of dimensions. The remaining errors (misclassifications within the roll-call data) are attributed to noise or some other unknown source. Thus, the eigenvalue and elbow rules are not tests in any strict sense. They provide guidance for researchers regarding when adding additional dimensions will reduce the number of errors sufficiently.

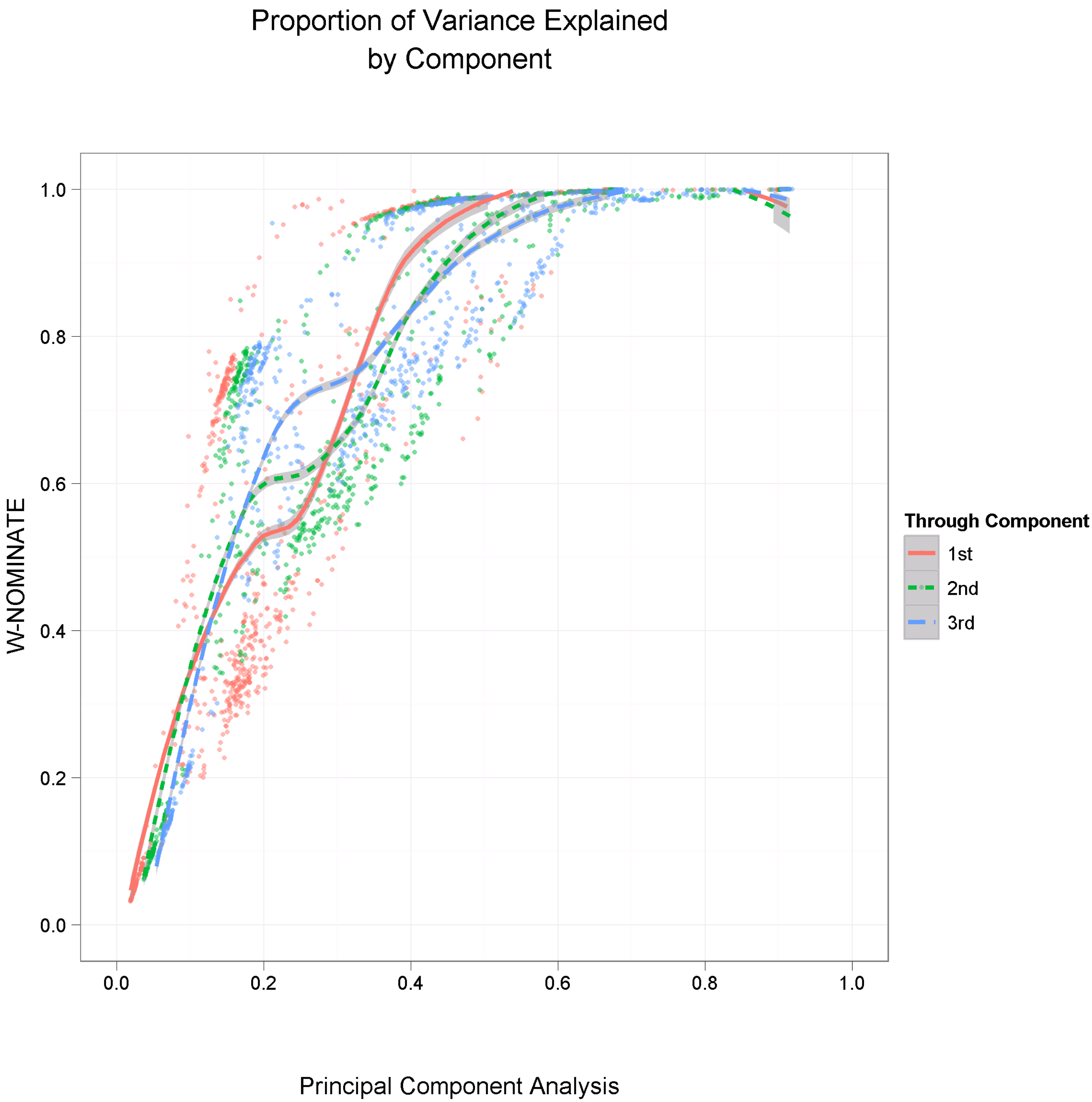
In the simulations below, we focus on Kaiser's eigenvalue-greater-than-one rule. For any given roll-call matrix, we infer that the latent dimension space is equal to the number of eigenvalues greater than one. One reason for this decision is practical. In our parameter sweep, we generate $132,480$ simulations, making the visual inspection of scree plots impractical. A second reason is that the eigenvalue rule is a more conservative test. Our argument is that it is too *easy* to produce roll-call matrices that would lead researchers to conclude that there are few dimensions. Thus, we use the method that is most likely to identify the highest number of dimensions in any situation.

In all of the analyses below, we focus primarily on the results from a simple principle components exploratory analysis.[6] The reason for this is almost entirely computational, and in future versions of this paper we will include a full array of W-Nominate estimates. However, we note that our results are extremely unlikely to be sensitive to the scaling method deployed. In previous work, we have shown that principle components never does worse and usually does better than W-Nominate in recovering the correct number of latent dimensions.

Moreover, Figure 2 compares the two methods for a random sub-sample of the parameter space we explore below. The plot shows the percent of the overall variance in the roll-call matrices explained by the first, second, and third dimensions in both the principal components analysis (PCA) and W-Nominate. The plot shows that the WNOMINATE procedure always accounts for *more* variance with these first three dimensions than does PCA. Thus, in the context of our research questions, using PCA is a conservative approach.

[6]We use the `princomp()` command in the `stats` package of R v2.11.

Figure 2: Dimensionality resulting from principle components analysis and WNominate



Poole and Rosenthal's WNOMINATE method always results in a lower number of dimensions than principle components analysis.

3 Spatial Voting

With these technical matters behind us, it is now possible to turn to the Monte Carlo simulations themselves. In each computational simulation, we generate the ideal points of members from a known distribution in a space with a known number of dimensions. We generate observations from these ideal points similar to roll calls and then apply scaling techniques to this data to answer two questions. First, under what circumstances (i.e., for what parameter settings) do we recover the correct number of dimensions from the simulated data? Second, under what circumstances do we recover the kinds of low-dimensional solutions common to analyses of real-world congresses? That is, we have a series of premises we think might be true. We observe a set of consequence, q, of those premises in our simulations. We apply a scaling procedure to those q's and derive a space. Is that space similar to the one we started with? If not, does it resemble the kinds of space we observe in estimates based on real-world data?

3.1 Basic spatial voting

In the first set of simulations, we focus on the very simplest of spatial voting models. In all the simulations in this paper we fix the number of members to 100 to be identical to the U.S. Senate. Member preferences, denoted x_i, are drawn from a multivariate normal distribution $x_i \sim N(\mathbf{0}_p, \mathbf{I}_{p\times p})$ where p is the assumed dimensionality of the space.

For each simulation we generate N observations for each member. That is, we ask members to cast a vote comparing a single status quo, a_j, and a single proposal, b_j. Members vote for the alternative that minimizes their squared error loss. That is,

$$y_{ij} = \begin{cases} 0 & \text{if } \|x_i - a_j\|^2 - \|x_i - b_j\|^2 < 0 \\ 1 & \text{if } \|x_i - a_j\|^2 - \|x_i - b_j\|^2 > 0 \end{cases} \tag{1}$$

To offer a fair test, it is necessary to generate status quo points that result in cut-points (or separating hyperplanes) spread evenly throughout the space occupied by members. We therefore use the following procedure. First, we randomly select (with replacement) one member to be the proposer, whom we assume proposes her own ideal point. Second, we randomly draw a "cut-point", c_j, from the distribution $c_j \sim N(\mathbf{0}_p, \frac{1}{2}\mathbf{I}_{p\times p})$. We then project across this cut-point to specify a status quo. That is, the status quo position on dimension p is chosen as:

$$a_{jp} = \left(\begin{array}{ll} c_{jp} - |c_{jp} - b_{jp}| & \text{if } b_{jp} > c_{jp} \\ c_{jp} + |c_{jp} - b_{jp}| & \text{if } b_{jp} < c_{jp} \end{array} \right. \tag{2}$$

Thus, in these simulations, we vary only two parameters. First, we vary the number of dimensions, p, from 1 to 10 by increments of 1 and then from 10-100 by increments of 10.[7] Second, we vary the number of roll calls. We consider simulations with 400, 800, and 2000 votes. The U.S. Senate averages fewer than 1,000 roll-calls, so the last represents far more data than we usually observe.

Each simulation results in a roll-call matrix. We then conduct a principal components analysis of each matrix and tabulate the number of eigenvalues greater than one. This is our estimate for the number of recovered dimensions. Figure 3 shows the results from these simulations.

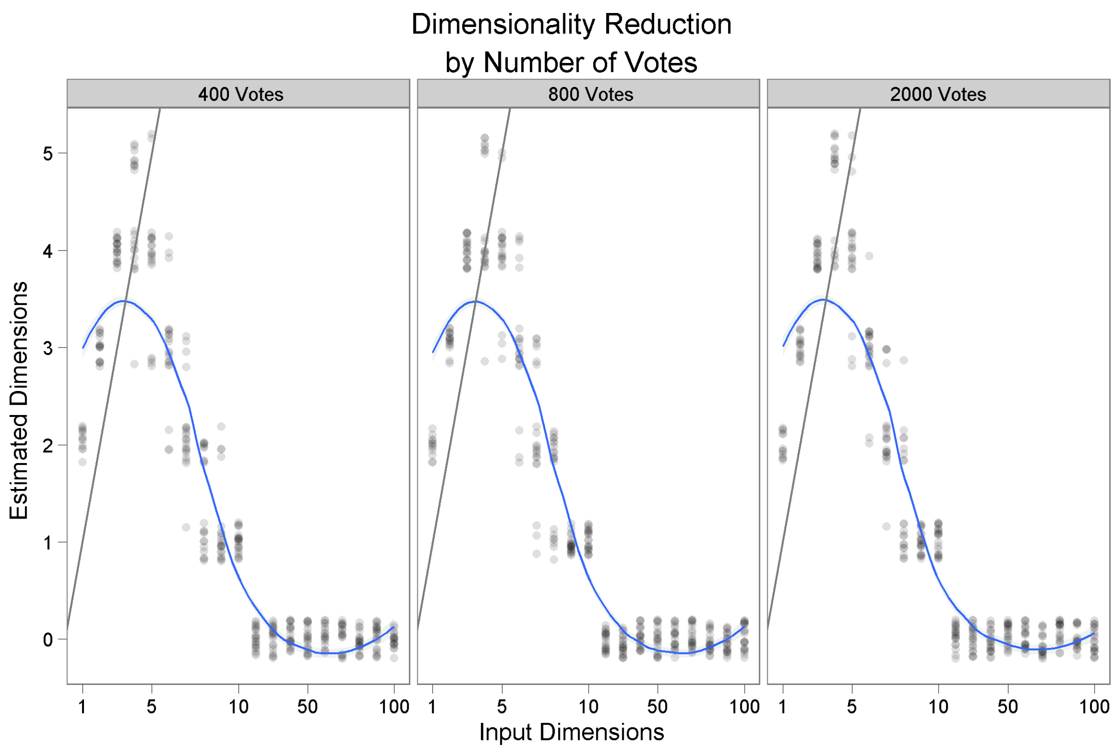
Figure 3 plots the number of estimated dimensions against the number real dimensions for approximately 2,400 separate simulations at each parameter setting. If the procedure were recovering the correct number of dimensions, all of the points would be at or near the straight gray line. However, the number of dimensions (p) increases, PCA begins to return far fewer dimensions than we put in. For moderately high settings ($p = [6, 10]$) the procedure returns the something in the range of the "one-and-a-half" dimensions. For higher settings, however, no dimensions are returned, as at these levels, there is insufficient data to identify anything but noise.

3.2 Bi-modal preference distributions

Next, we consider the possibility that member preferences might be distributed in some bimodal form. We assume that member preferences are drawn from two multivariate normal distributions (a mixture of normals). This mimics, in a minimal fashion, the effect that party institutions, pri-

[7] Thus the vector of parameters considered for p is $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100)$.

Figure 3: Real versus recovered dimensions for the basic spatial voting model



The points represent the number of real versus recovered dimensions that were identified by the simulation. The straight gray line shows the location at which the points should be if the procedure were recovering the "correct" number of dimensions. The curved blue line is a loess line to aid interpretation.

maries, or activists might have on member positions in some policy space (Aldrich 1983; Aldrich and McGinnis 1989; Montgomery 2010). The variance-covariance matrix remains fixed within each normal distribution as above.

In these simulations we add a new parameter, D, that indicates the distance between the means of the two distributions. We begin by assuming that the distribution means are separated along *each* dimension. Thus, if the multivariate mean of one population is the vector [.5, .5], then the mean of the second population would be [-.5, .5] and D would be 1.[8] Of course, our earlier results are equivalent to the case where $D = 0$

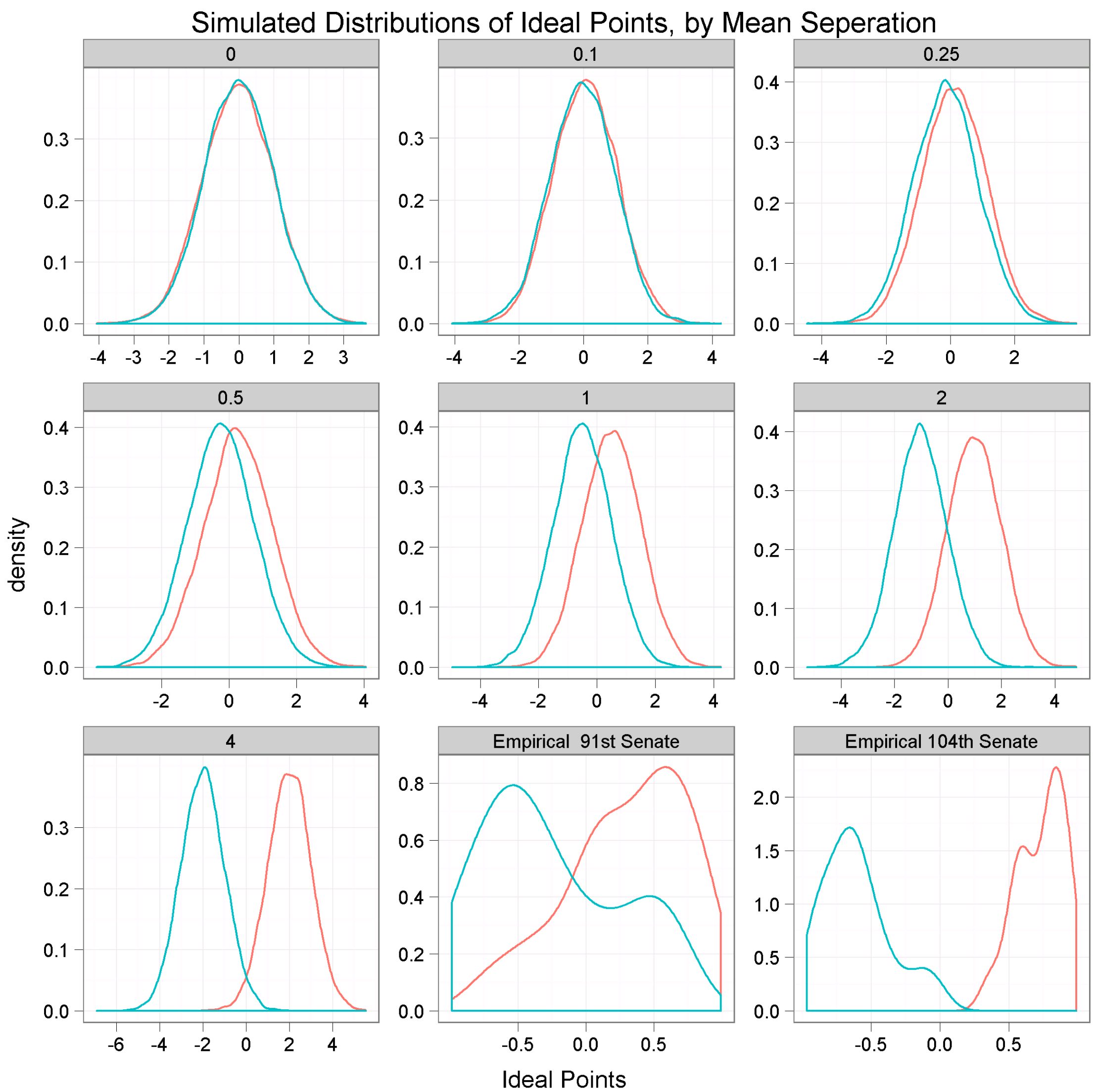
Figure 4 shows examples of these bimodal distributions for various possible parameter settings for D. The final panels of Figure 4 also show the distribution of the members of the 91^{st} and 104^{th} Senates. The figure demonstrates that the level of bimodality we consider in these simulations is no greater than what we observe in the contemporary congress.

Figure 5 plots the number of actual dimensions against the number of dimensions with eigenvalues greater than 1.0 for varying settings of the separation parameter D. In our simulations, we consider values of $D = (0, 0.25, 0.5, 1, 2, 4)$. As can be seen, the large values for D have a significant impact on the ability of PCA to recover the correct dimensionality. This is true even when the actual number of dimensions is fairly low (e.g., 5). In general, if the distribution of ideal points is actually bi-modal (with identical variances), then as those two distributions begin to show any separation (as the means move apart), PCA collapse almost immediately to the recovery of one-plus dimensions. This is especially true when the degree of polarization is similar to what we observe in the contemporary House and Senate. In these situations (shown in the right panels of Figure 5) the one-plus dimensionality result is almost all that is ever recovered.

One concern with these results is that by moving the two distributions apart on *every* dimension, we are implicitly creating a new dimension that contains most of the variance. An alternative approach would be to move the parties apart on only a subset of the total dimensions. Thus we can

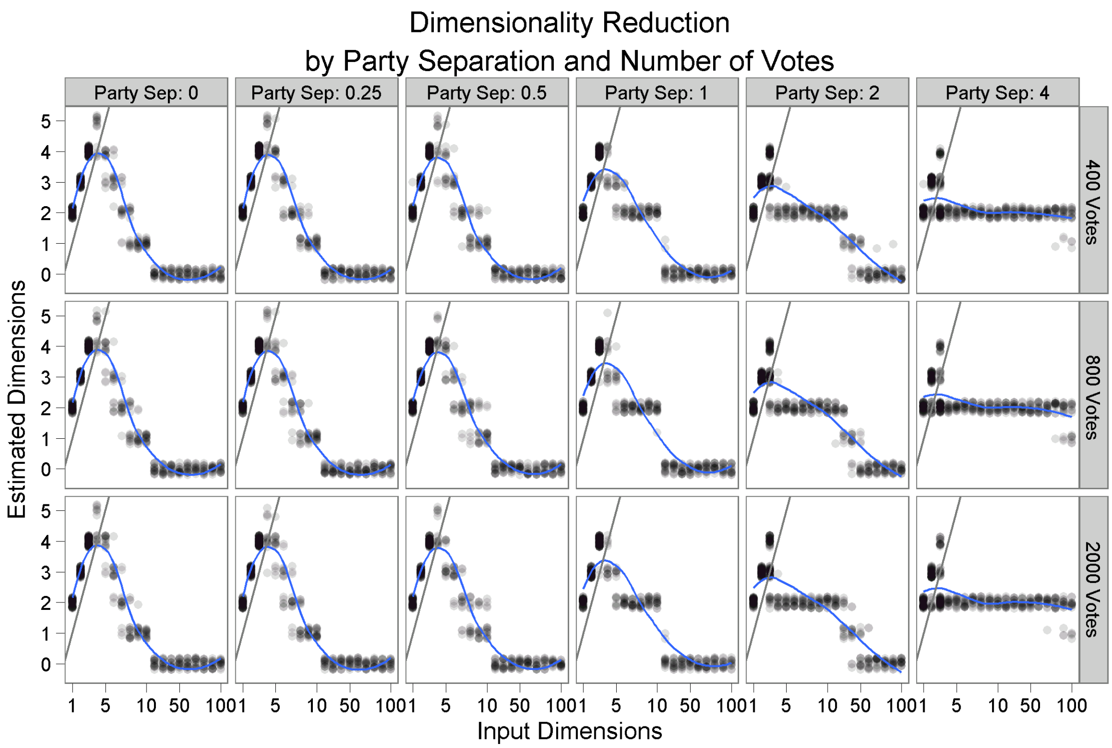
[8] In these simulations we also add an additional parameter, M, which indicates the size of the majority party. We include simulations where the majority is 51 members and 60 members. However, there is no evidence that this parameter has any influence on dimensionality in our simulations, and we do not discuss it further.

Figure 4: Visualizing distribution separation



The first seven panels show a random draw of 10,000 observations from the bimodal distribution used in our simulations. The final two panels show the distribution of 1^{st} *dimensional WNOMINATE scores for the* 91^{st} *and* 104^{th} *Senates.*

Figure 5: Real versus recovered dimensions for the basic spatial voting model by party separation



The points represent the number of real versus recovered dimensions that were identified by the simulation. The straight gray line shows the location at which the points should be if the procedure were recovering the "correct" number of dimensions. The curved blue line is a loess line to aid interpretation.

add a new parameter, $p_D \in [0, 1, \ldots, p]$, that indicates the number of separation dimensions. That is, it is the integer indicating the number of dimensions on which the two distributions differ.[9]

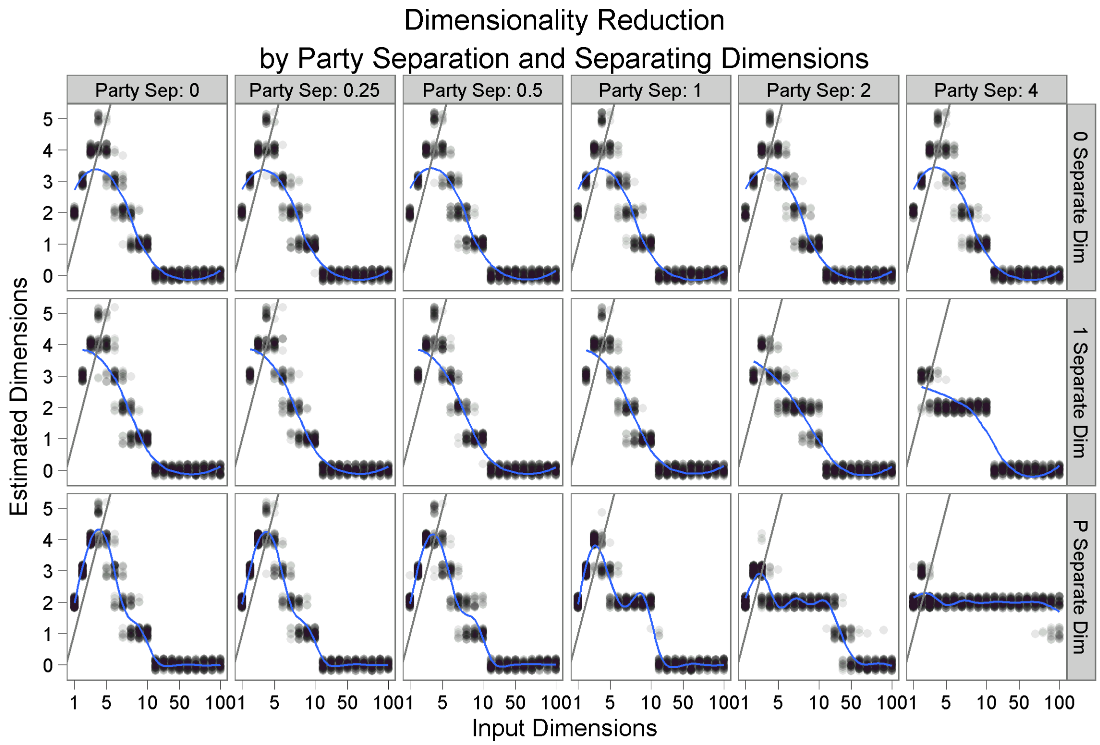
Figure 6 shows the results when $p_d = (0, 1, p)$. The figure shows that we are more likely to recover a lower number of dimensions for any degree of party separation as the number of separation dimensions, p_d, increases. Note for any level of party separation, the number of dimensions decreases as p_d increases. In addition, the middle row of panels in this figure shows the results when there is separation along only a single policy dimension ($p_d = 1$). This row of figures shows that, given the degree of "polarization" we observe in the contemporary congress (i.e., $D = 4$), we will never recover more than three dimensions even if the distributions are separated along only a single dimension. Indeed, in this situation ($p_d = 1$ and $D = 4$), we recover either one, two, or zero significant dimensions, regardless of the number of dimensions there actually are.

4 Distributive politics

So far we have focused on simulations where the assumed behavior of members is closely aligned with what is commonly assumed by PR and other scaling procedures. In this section, we turn to a very different set of assumptions for member behavior. Besides the standard spatial account, perhaps the most prominent class of models are the bargaining models first popularized by Baron and Ferejohn (1989). In these models, members are not seeking to extract policy benefits, but rather to "divide the dollar" between themselves. Members simply seek to form coalitions with whom they can share some common pot of resources. One of the main tasks of these models is to understand how these resources will be distributed, but for our purposes this question is irrelevant. We simply assume that those members who are included in the winning coalition will support the bill and those who do not benefit from the division of resources will oppose the bill.

[9] We ran simulations in which the two distributions differ on 0,1,2,3, and p dimensions. However, there were not substantial differences between the results for 1,2, and 3. We therefore only show the results for 0, 1, and p.

Figure 6: Real versus recovered dimensions for the basic spatial voting model by the number of dimensions on which parties are separated



The points represent the number of real versus recovered dimensions that were identified by the simulation. The straight gray line shows the location at which the points should be if the procedure were recovering the "correct" number of dimensions. The curved blue line is a loess line to aid interpretation.

4.1 Basic distributive politics

In the basic variant of the model, we assume that members form a random coalition of size $\alpha \in [51, 100]$. One member, the proposer, randomly chooses a subset of Congress of size α. These chosen members are included in the division of resources and vote 'yea', and the remainder vote 'nay.'

More formally, we implement this model as follows. For each roll call j, each member i is assigned a random number, r_{ij}, drawn from the standard normal distribution.

$$r_{ij} \sim N(0, 1). \tag{3}$$

The proposer then ranks members based on their value of r_{ij} and awards the highest α members a distributive benefit B_{ij}.[10]

$$B_{ij} = \left(\begin{array}{ll} 1 & \text{if } r_{ij} > rank_{\alpha}(r_j) \\ 0 & \text{if } r_{ij} < rank_{\alpha}(r_j) \end{array} \right. \tag{4}$$

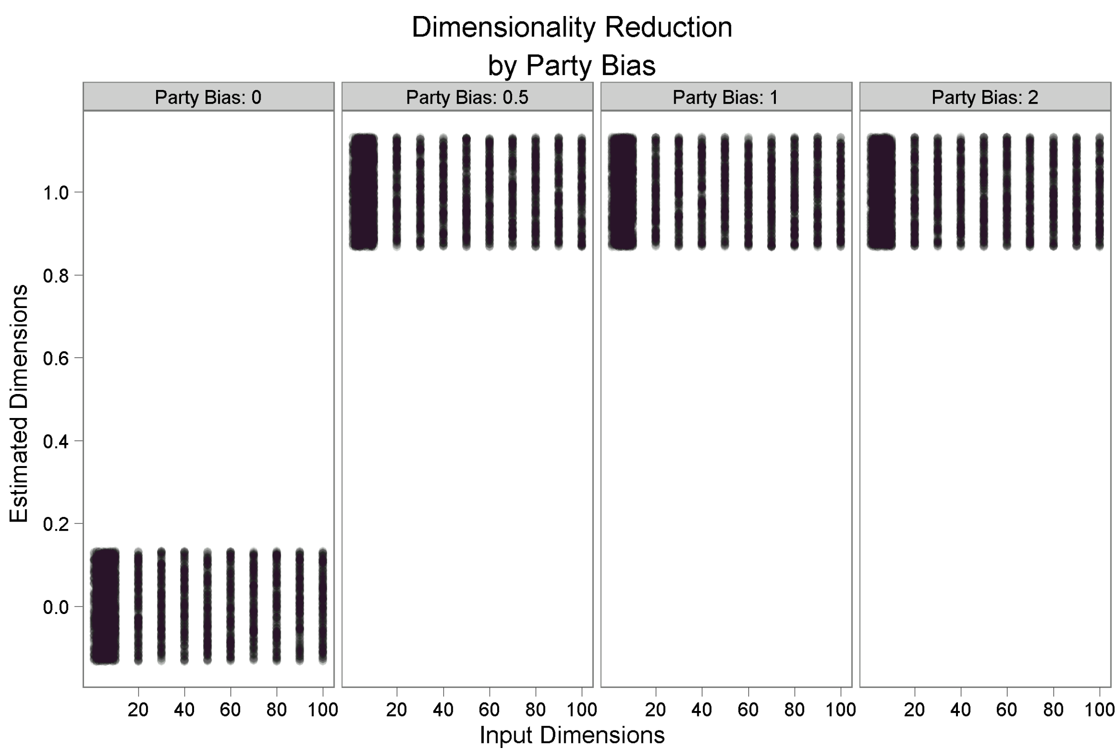
Finally, members who receive the distributive benefit always vote in favor of the bill, while the remainder vote 'nay.'

$$y_{ij} = \left(\begin{array}{ll} 1 & \text{if } B_{ij} = 1 \\ 0 & \text{if } B_{ij} = 0 \end{array} \right. \tag{5}$$

Substantively, this model indicates that there are basically 99 separate dimensions members care about. They care whether they personally are included in the coalition and care nothing about what other members are included. From the perspective of a standard scaling technique, however, this is random voting. Our expectation is that all scaling techniques would find no significant dimensions. And, indeed, the left hand panel of Figure 7 shows exactly that.

[10] In the simulation code, this is denoted as "Bribes."

Figure 7: Real versus recovered dimensions for the distributive politics model



The points represent the number of real versus recovered dimensions that were identified by the simulation.

4.2 Distributive politics with a partisan twist

Next, we implement a simple twist to above setup. We suppose that MCs are members of two political parties and that they are somewhat biased in towards co-partisans as they form these coalitions. Formally, we now assign each member a new trait τ_i that indicates whether they are in party Ψ or party Θ. There are M_θ members in party Θ and $M_\psi = (100 - M_\theta)$ in party Ψ.[11]

For each roll call j we randomly choose one member k to form the coalition. In this case we alter Equation 3 such that r_i is assigned to each member $i \neq k$ from the distribution:

$$r_{ij} = \begin{pmatrix} N(\rho, 1) & \text{if } \tau_k = \tau_i \\ N(-\rho, 1) & \text{if } \tau_k \neq \tau_i \end{pmatrix} \quad (6)$$

where ρ is the parameter controlling intra-party bias.

Just as before, the members are ranked based on their value of r_{ij}, and the highest α members are included in the coalition and receive the benefit (Equation 4) and members vote based solely on the inclusion in the coalition (Equation 5). Thus, as ρ increases in value, co-partisans vote in the same way on an increasing number of roll calls.

Figure 7 shows the results of the simulation. The left panel shows the number of recovered versus real dimensions when there is no party bias. The panels to the right show this same information for increasing values of ρ. As can be seen, PCA returns exactly one dimension when there is some party bias and exactly zero when there is none. Moreover, there is no differentiation between the different values of ρ.

5 Mixing distributive and spatial motivations

Both the spatial and distributive models of politics, however, are by themselves incomplete. Scholars have long argued that many factors simultaneously affect the voting decisions made by members

[11] Just as in the bimodality simulations above, the size of the majority makes no difference to our results, nor does the size of the coalition being formed.

of Congress (c.f., Mayhew 1974; Truman 1971). Senators' votes on the 2009 Patient Protection and Affordable Care Act, for example, were certainly informed by ideological preferences, but there was also a distributive component, seen most clearly in the federal reimbursement for expenses received by the state of Nebraska as part of the Democrats' attempts to gain Senator Ben Nelson's vote. In our final set of simulations, we examine the effect that mixing these factors may have on estimates of dimensionality in roll-call data.

As a first cut, we specify a voting heuristic for members that cleanly combines both spatial and distributive motives. We introduce a new parameter, $\gamma \in [0, 1]$, that controls the degree to which roll-call votes are made based on each concern. Just as in Section 3.1, each roll-call j is associated with a randomly selected status quo, a_j, and proposal point b_j. Member preferences are drawn from the multivariate normal distribution n $x_i \sim N(\mathbf{0}_p, \mathbf{I}_{p\times p})$. In addition, just as in Section 4, for each roll-call there is a distributive element. Members are chosen at random to receive a distributive benefit from the proposed bill.

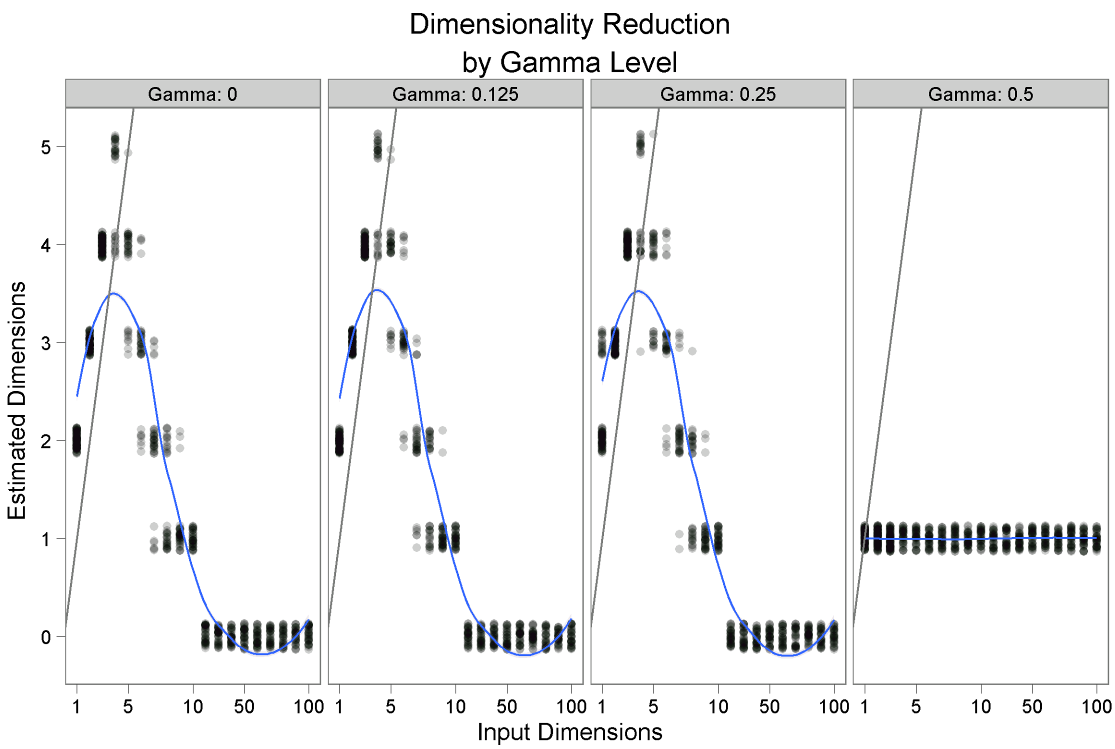
To cleanly combine these two competing forces, we re-scale the spatial component using the cumulative distribution function of the standard normal distribution, $\Phi(.)$, similar to a probit link function. We can then use the γ parameter to weight between distributive and spatial concerns. Formally, this is denoted as:

$$y_{ij} = \begin{cases} 0 & \text{if } (1-\gamma) \times \Phi\left(\|x_i - a_j\|^2 - \|x_i - b_j\|^2\right) + \gamma \times B_{ij} < .5 \\ 1 & \text{if } (1-\gamma) \times \Phi\left(\|x_i - a_j\|^2 - \|x_i - b_j\|^2\right) + \gamma \times B_{ij} > .5 \end{cases} \tag{7}$$

We generate roll-call matrices using this heuristic with five values of $\gamma = (0, 0.125, 0.25, 0.5, 1)$. Obviously, any values of 0.5 and greater are equivalent to pure distributive voting. However, this would not be true if we allowed MCs to vote probabilistically. The results of these simulations are shown in Figure 8, which depicts a subset of the results graphed in 9. The figure shows that for values of γ below the 0.5 threshold, the number of recovered dimensions is only modestly affected by the addition of impartial distributive politics.

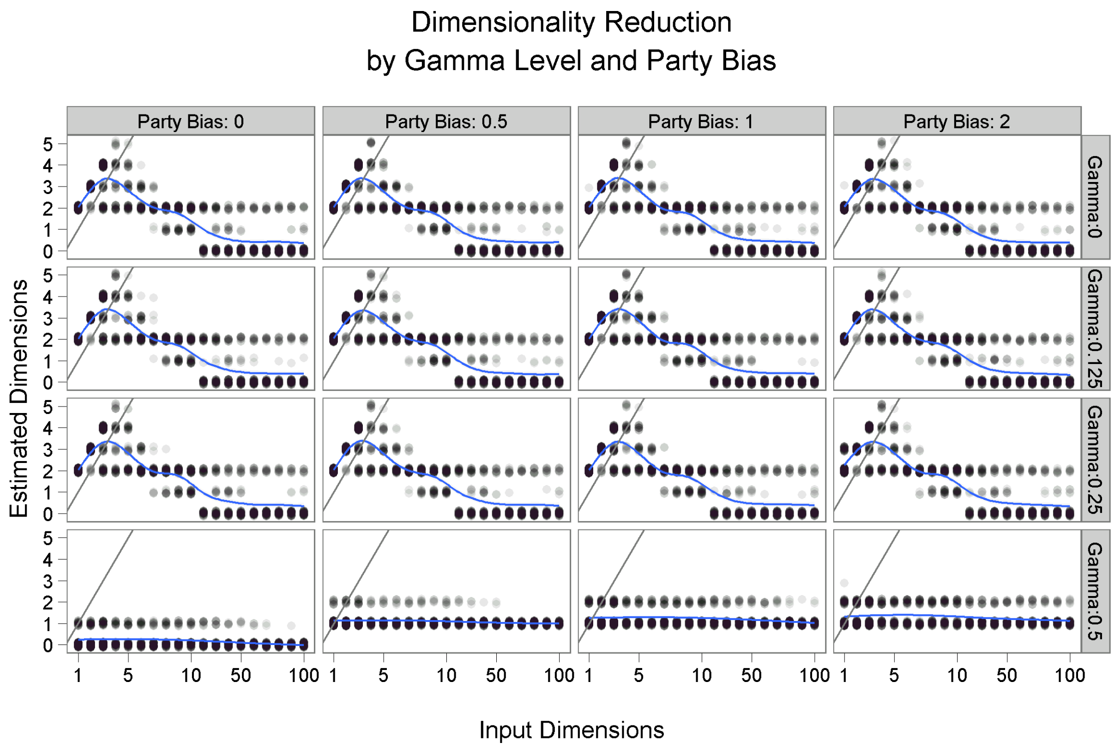
In our final set of simulations, we consider the impact of allowing for different levels of co-

Figure 8: Real versus recovered dimensions when mixing policy and distributive motives with no partisan bias in distributive benefits



The points represent the number of real versus recovered dimensions that were identified by the simulation. The straight gray line shows the location at which the points should be if the procedure were recovering the "correct" number of dimensions. The curved blue line is a loess line to aid interpretation.

Figure 9: Real versus recovered dimensions when mixing policy and distributive motives by partisan bias in distributive benefits



The points represent the number of real versus recovered dimensions that were identified by the simulation. The straight gray line shows the location at which the points should be if the procedure were recovering the "correct" number of dimensions. The curved blue line is a loess line to aid interpretation.

partisan bias in the distributive side of things. That is, we assume that random assignment to participate in a coalition follows Equation 6 above. The results from these simulations are shown in Figure 9. The simulations reveal that there is not much interaction between these two parameters, except in certain important edge cases.

6 Discussion and Conclusion

> "Parties ... help to map complex issues ... into a low-dimensional space" (Poole and Rosenthal 2007, p.43).

We will conclude by noting two important implications of this work. To begin with, these results speak to the need for expanded attention to models of Congress (and politics more generally) that are robust to assumptions about the number of dimensions. There is a considerable difference in what spatial models say about politics if the space is or is not *exactly* one-dimensional. In one dimension there is a median voter. If, however, the space is perturbed even infinitesimally away from a pure single dimension, there is no median, and a great many results evaporate.

While not strictly requiring a single dimension, nearly all applications of Romer-Rosenthal agenda setting are based on just as exacting a unidimensionality assumption for the very same reason that these results nearly always need a median voter to exist (Romer and Rosenthal 1978). Pivot point models are the same category (e.g., Krehbiel 1998). Some of its results fall apart just as completely as the median voter result.

Many derivations of Duvergerian style results, prominent models of elections and government formation under proportional representation (Austen-Smith and Banks 1988), informational models of Congress (Gilligan and Krehbiel 1989; Krehbiel 1991), Perrson-Tabelini models (Persson and Tabellini 2002), and others (e.g., Iverson and Soskice 2001) require a very exacting form of unidimensionality. Many, if not all, of their derivations simply disappear if the assumption fails to the slightest possible degree (Kramer 1973). Even many of the results that are used to study n-dimensional policy spaces are built on repeated application of median voter-based logics (Shepsle and Weingast 1987; Laver and Shepsle 1990).

Thus far, we have focused exclusively on the number of dimensions. However, there are additional features of the basic space of political competition. A second question many who use PR results would like to answer is what the dimensions actually are. In the case PR scaling, it is quite common to assert that the dimensions are ideological, and that the first dimension is what we ordinarily mean by the liberal-conservative ideological dimension.

This may be true, but the problem is exactly the same here. It could be that the major dimension estimated is a liberal-conservative dimension, but it could just as easily be something else. The spatial model is no help. Its generality is precisely such that the dimensions of choice are not defined within the theory. It must be assumed, asserted, or derived in some other fashion. This is a problem that is at least as well known in scaling methods as the determination of the appropriate number of dimensions.

They are left to the researcher to interpret, whether in factor analysis, PR scaling, or whatever else. Our point here is not only to remind the reader of these well known considerations, but to remind the reader that the theory in which most applications of scaling rely is not a source for addressing these questions. The theory is one of choice, not a theory of the nature of preferences.

Consider the often made claim that the first dimension is a liberal-conservative ideology, and that it is those preferences that are the most important causes of voting choices (Poole and Rosenthal 2007). Thus, the very clear pattern of increased polarization in the Congress is interpreted as ideology leading to greater party polarization and thus higher levels of party voting. While that set of causal claims is consistent with the observed patterns in the scaled roll-call voting data, so is the causal claim that parties have become stronger and more unified, and they have led their members to vote more along party lines.

Thus, the first dimension of PR captures this enhanced coherence of Democrats voting with Democrats, Republicans voting with Republicans, and the estimated first dimension is the line of cleavage along which the parties divide.What we (and the media) call liberal and conservative is what divides Democrats from Republicans, that which they have chosen to reveal in their public discussions and on the floor of Congress. It did not used to include civil rights, but now it does. It did not used to include abortion, but now it does. This may simply be a result of the changing position of the parties rather than any more fundamental change in the relationship between policy debates on traditional economic issues and abortion.

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A Appendix

Abridged code for the simulation

```
####################
# Define Functions #
####################

PreferenceGenerator <- function(Cuts = FALSE){
  Party <- c(rep(1, MajorityPartySize), rep(-1, NObservations - MajorityPartySize))
  Alpha1 <- c(rep((PartySeparation/2) * 1, NSeparateDimensions),
   rep(0, NDimensions - NSeparateDimensions))
  Alpha2 <- c(rep((PartySeparation/2) * -1, NSeparateDimensions),
   rep(0, NDimensions - NSeparateDimensions))
  if (Cuts == FALSE){
    NormalDataPrefs2 <- rmvnorm(NObservations - MajorityPartySize, Alpha2,
     diag(NormalVariance, NDimensions))
    NormalDataPrefs1 <- rmvnorm(MajorityPartySize, Alpha1,
     diag(NormalVariance, NDimensions))
    }
  if (Cuts == TRUE){
    NVotes.Maj = floor(NVotes*MajorityPartySize/NObservations)
    NVotes.Min = NVotes - NVotes.Maj
    NormalDataPrefs1 <- rmvnorm(NVotes.Maj, Alpha1,
     diag(NormalVariance/Cut.Squeeze, NDimensions))
    NormalDataPrefs2 <- rmvnorm(NVotes.Min, Alpha2,
     diag(NormalVariance/Cut.Squeeze, NDimensions))
    }
  NormalData <- data.frame(rbind(NormalDataPrefs1, NormalDataPrefs2))
  if(Cuts == FALSE){ NormalData$party <- Party }
  return(NormalData)
  }

DistanceDifferencer <- function(x){
  Temp1 <- (rowSums((as.matrix(IdealPointArray) - matrix(c(Proposals[x,]),
   nrow=NObservations, ncol=NDimensions, byrow=T)) ^ 2))
  Temp2 <- (rowSums((as.matrix(IdealPointArray) - matrix(c(StatusQuoPoints[x,]),
```

```
   nrow=NObservations, ncol=NDimensions, byrow=T)) ^ 2))
  return(Temp1 - Temp2)
  }

Bribe <- function(x){
  Give.Temp <- rnorm(NObservations, IdealPoints$party *
   DistributivePartyBias * ProposerParty[x] ,1)
  return((rank(Give.Temp) <= NBribes)*1)
  }

##############################
# Set of Parameters to Sweep #
##############################

# For example:

Sweep.NDimensions <- c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100)
Sweep.NVotes <- c(400, 800, 2000)
Sweep.NObservations <- c(100)

Sweep.NormalMean <- 0
Sweep.NormalVariance <- 1
Sweep.NSimulations <- 1:1

Sweep.NSeparateDimensions <- c(0, 1, 2, 3, 999)
Sweep.PartySeparation <- c(0, 1/4, 1/2, 1, 2, 4)

Sweep.MajorityPartySize <- c(51, 60)

Sweep.Gamma <- c(0, 1/8, 1/4, 1/2, 1)

Sweep.DistributivePartyBias <- c(0, 1/2, 1, 2)
Sweep.NBribes <- c(51, 60)

Sweep.Cut.Squeeze <- c(2)  # Aids in narrowing of SQ space

# ...Expand all combinations of parameters to
# a matrix, SweepParameters...

####################################
# Iterate over the parameter space #
####################################

for (i in 1:n.sims){
```

```
# ...Assign parameters to local variables...

  # Generate ideal points
  IdealPoints <- PreferenceGenerator()
  IdealPointArray <- as.matrix(IdealPoints[, 1:NDimensions])

  # Select a random proposer, their ideal point, and party
  WhoProposes <- sample(c(1:NObservations), NVotes, replace=T)
  Proposals <- as.matrix(IdealPoints[WhoProposes, 1:NDimensions])
  ProposerParty <- IdealPoints$party[WhoProposes]

  # Draw cut-points from the same distribution as the voters.
  CutPoints <- PreferenceGenerator(Cuts=T)[,1:NDimensions]

  StatusQuoPoints <- (CutPoints - abs(CutPoints - Proposals)*(Proposals > CutPoints) +
   abs(CutPoints - Proposals)*(Proposals < CutPoints))
   StatusQuoPoints <- as.matrix(StatusQuoPoints)

# Given the proposer's ideal point and the cutpoint,
# we can infer backwards to the status quo position.

  # Ideological component of voting probability function
  IdeologyPart <- (1 - pnorm(DifferenceinDistances))

  # Distributive
  DistributivePart <- sapply(c(1:NVotes), Bribe)

# Combined Voting Probability Function

  VoteProbability <- (1 - Gamma) * IdeologyPart + Gamma * DistributivePart
  VoteProbability[VoteProbability > 1] <- 1
  VoteProbability[VoteProbability < 0] <- 0
  VoteProbability<-matrix(VoteProbability, nrow=NObservations, ncol=NVotes)

  # Translating voting probability into Yeas and Nays
  RandomVotes <- t((VoteProbability > 1/2) * 1)

# ...Perform any analyses / save any output...
  }
```